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A mixed integer programming based approach for unit commitment problem

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ABSTRACT

Unit commitment (UC) problem is a challenging task in power system operation that has attracted much attention in the two last decades. It aims to find the optimum statues of the thermal units and their optimum to the predicted load demands in order to minimize the total production cost. Within this context, this paper presents a piecewise linear approximation method for solving this mixed integer problem (MIP). Power balance, generation capacity, minimum up/down times and spinning reserve constraints are considered in this study. The proposed method is implemented in GAMS 24.2. Simulation results are carried out using the ten-unit system.

1. Introduction

The unit commitment (UC) problem is a nonlinear problem which considers two sub-problems. It aims to determine the ON/OFF statues of generating units and to schedule the outputs for all committed generators, for a given horizon time. The UC problem can be formulated as an optimization problem, where the objective function is the total production cost (Saravanan et al., 2016). The total operation cost of thermal units comprises the generation cost, the stat-up cost and the shut-down cost. Start-up cost is the cost generation before the generator is committed. It corresponds to the cost of the fuel needed to meet the required steam conditions (Wood and Wollenberg, 2012). The shut-down cost is associated with the gradual reduction of the thermal unit from the nominal minimum power to the actual stop of the unit. Generally, shut-down costs are much smaller than start-up costs; hence several works have neglected them (Tuffaha and Gravdahl, 2013; Wood and Wollenberg, 2012). The decision vector involved in the UC problem comprises the status and the output of each unit (Wood and Wollenberg, 2012). The first one is a binary value; however, the second one is a real number. Thus, the UC problem can be considered as a mixed-integer problem (MIP). In the past, the most techniques proposed for solving the UC problem are based on the Lagrangian relaxation (Gubina and Strmcnik, 1991; Murata and Yamashiro, 2005; Virmani et al., 1989). However, the effectiveness of these techniques degrades with the number of units. Other techniques such as, priority list method (Senjyu et al., 2003), dynamic programming (Singhal and Sharma, 2011) were frequently used in the UC problem.

In Carrion and Arroyo (2006), an optimization technique to solve MILP-UC using a set of binary variables has been presented. The study has demonstrated the relationship between binary variables and the computational time. The binary variables have been used in the previous study to present the generation status and to determine the start-up and shut-down costs. The study also has explained how the increase of the binary variables would effectively increase the number of constraints and eventually increase the modeling capabilities.

The dramatic appearance of modern software, such as the general algebraic modeling system (GAMS), has made mixed integer programming an attractive alternative for solving the UC problem. The first formulation of the UC as a MIP is presented in Garver (1963). In recent years, some works, have concentrated on finding more efficient mixed-integer programming based models for the UC problem (Ostrowski et al., 2012). Generally, the most of these models have been approximated by linear models (Lopez et al., 2012). Several techniques used for the linearization of the quadratic fuel cost and the start-up cost have been reported in Frangioni et al. (2009). In Carrion and Arroyo (2006), a piecewise linear (PWL) approximation technique was proposed to linearize the cost function of the UC. A mathematical

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approach has been proposed in Wu (2011) in order to find the optimal PWL of the quadratic cost function.

In this paper, a technique based on the PWL is used to solve the UC problem. The cost function is minimized subject to several dynamic constraints such as, power balance constraint, generation capacity and minimum up/down times. The obtained mixed-integer linear programming (MILP) is implemented in GAMS 24.2 (Alqunun and Crossley, 2016). The quadratic cost function is divided into high number of segments in order to ensure accuracy and feasibility.

2. Problem statement

2.1. Objective function

In the most research works, the UC problem was defined as minimization problem (Carrion and Arroyo, 2006). It aims to find the on/off status of units as well as the optimal schedule of generating outputs according to the variation of power demands during a certain time period. The objective function to be minimized is the total generation cost, which can be expressed by the following equation (Wang and Singh, 2009). As given in in equation (1), the total generation cost is the sum of the total fuel cost, the total start-up cost and the total shut-down cost. To simplify the problem, the shut-down cost can be neglected.

$$C_T = [\sum_{t=1}^{T}\sum_{i=1}^{N} C_i(P_i^t)]u_i^t + [\sum_{t=1}^{T}\sum_{i=1}^{N} S_i^t]u_i^t(1-u_i^t) + [\sum_{t=1}^{T}\sum_{i=1}^{N} D_i^t]u_i^t(1-u_i^t) \quad (1)$$

In this study, $= 24\ hours$. The fuel cost of unit i at time t is expressed by the following quadratic equation.

$$C_i(P_i^t) = a_i + b_i P_i^t + c_i (P_i^t)^2 \quad (2)$$

where a_i, b_i, c_i are the cost coefficients of the i-th unit.

The start-up cost S_i^t of unit i at time t, which is the cost for restarting the unit when it is OFF, can be expressed by an exponential, linear or two-valued staircase functions (Damousis et al., 2004). In this study, the last one function is used.

$$S_i^t = \begin{cases} S_{hi} & if\ T_{i,OFF}^t \le T_i^D + T_i^C \\ S_{ci} & if\ T_{i,OFF}^t > T_i^D + T_i^C \end{cases} \quad (3)$$

2.2. Problem constraints

In general four main constraints are taken into account in the UC problem.

Power balance constraint

At each time period t, the total power generation must cover the total demand power P_D^t plus the total transmission losses P_L^t. Thus, the power balance constraint can be described by the following equation.

$$\sum_{i=1}^{N}(P_i^t)u_i^t - P_D^t - P_L^t = 0 \quad (4)$$

Generation limits

Due to the unit design, the real power output of each unit i at hour t should be within its upper and lower limits.

$$P_i^{min} \le P_i^t \le P_i^{max} \quad (5)$$

Minimum up/down times

Minimum up/down times are the minimum OFF/ON durations of the unit before it can commutate to online/offline. These constraints are written as follows.

$$\begin{cases} T_{i,ON}^t > T_i^U \\ T_{i,OFF}^t > T_i^D \end{cases} \quad (6)$$

Spinning reserve constraints

At each interval time t, the spinning reserve constraint is represented by the following inequality.

$$\sum_{i=1}^{N} P_i^t u_i^t \ge P_D^t + SR^t \quad (7)$$

3. Simulation results

To show the effectiveness of the proposed approach for solving the UC problem, the ten-unit system that is very well used for this kind of problems, is suggested in this study.

The mixed integer linear programming is used in this paper to accumulate the hourly operation cost of ten generating units. MILP allows the implementation of high number of parameters, positive/negative variables, binary variables and system constraints. Furthermore, the execution time of the MILP is less as compared to the traditional methods. The objective function of the MILP must be linear; therefore the quadratic cost function of the generating units is converted into piecewise function as shown in Fig. 1. The quadratic cost function is divided into high number of segments to ensure accuracy and feasibility. The generating units contain specific power segments and cost segments based on their characteristics.

The test system data are given in Table 1. The variation of the load during one day is shown in Table 2.

3.1. Test case data

The single-line diagram of the ten-unit system is given in Fig. 2.

3.2. Results and discussion

The hourly demand of the 10-units is shown in Fig. 3. The optimal power supply of the 10 generation units is illustrated in Fig. 4. The optimal unit statues are tabulated in Table 3.

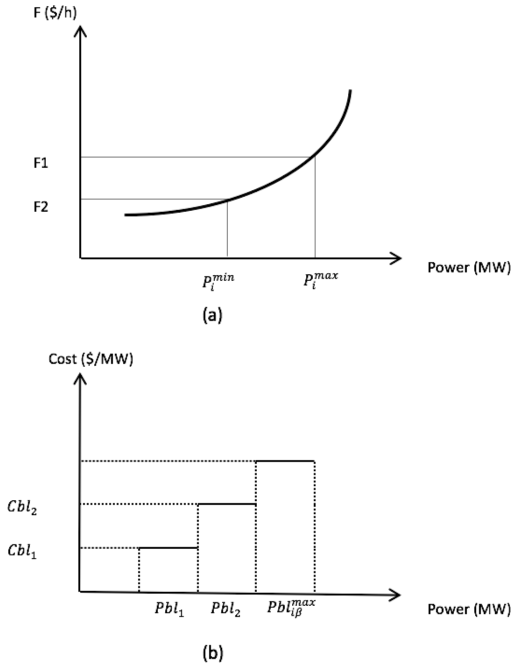

Fig. 1: Cost function (a) Quadratic curve, (b) Piecewise

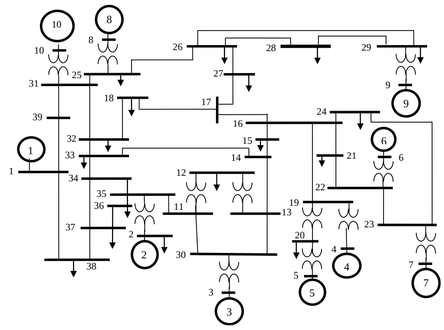

Fig. 2: Single-line diagram of the test system

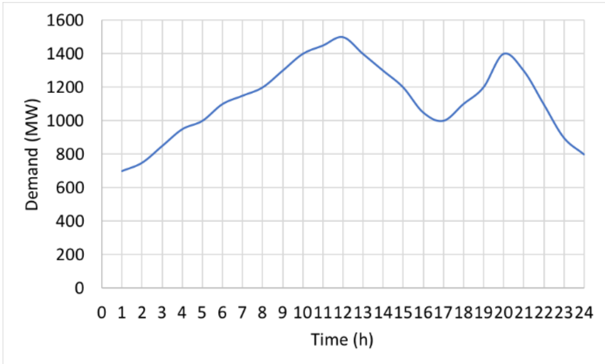

Fig. 3: Hourly demand of 10-unit system

Table 1: Unit data

Unit	P_i^{min}	P_i^{max}	a_i	b_i	c_i	T_i^U (h)	T_i^D (h)	S_{hi} ($)	S_{ci} ($)	T_i^c (h)	IS_i (h)
1	150	455	1000	16.19	0.00048	8	8	4500	9000	5	8
2	150	455	917	17.26	0.00031	8	8	5000	10,000	5	8
3	20	130	700	16.60	0.00200	5	5	550	1100	4	-5
4	20	130	680	16.50	0.00211	5	5	560	1120	4	-5
5	25	162	450	19.70	0.00398	6	6	900	1800	4	-6
6	20	80	370	22.26	0.00712	3	3	170	340	2	-3
7	25	85	480	27.74	0.00079	3	3	260	520	2	-3
8	10	55	660	25.92	0.00413	1	1	30	60	0	-1
9	10	55	665	27.27	0.00222	1	1	30	60	0	-1
10	10	55	770	27.79	0.00173	1	1	30	60	0	-1

Table 2: Load in MW for one day

Hour	1	2	3	4	5	6	7	8	9	10	11	12
Load	700	750	850	950	1000	1100	1150	1200	1300	1400	1450	1500
Hour	13	14	15	16	17	18	19	20	21	22	23	24
Load	1400	1300	1200	1050	1000	1100	1200	1400	1300	1100	900	800

Units 1 and 2 are committed all the time due to the largest capacity and lowest fuel cost amongst all the 10 generation units. Unit 5 is committed at hours 1-21, because the cumulative cost of the no-load, fuel and the start-up is less as compared to units 3 and 4. Unit 4 is operated immediately when the demand exceeds the maximum supply of units 1, 2 and 5, for example at hour 6. The reason of operating unit 4 at hour 6 is due to low fuel cost as compared to unit 3. It can be noticed that Unit 3 is off at hours 1-8, and started dispatching its maximum capacity of 130 MW at hour 9 when units 1, 2, 4 and 5 are no longer able to satisfy the demand.

From Table 1, it is clear that the maximum supply of units 1-5 is 1332 MW, however the demand exceeds this limit at hour 10. Therefore, unit 6 start operating at this hour since its fuel cost is less as compared to units 7, 8, 9 and 10. The dispatch from units 8 and 9 is only 3 hours during the 24-h period of time.

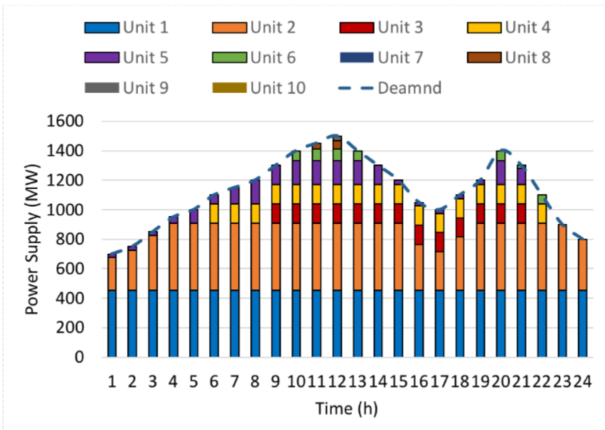

Fig. 4: Hourly power dispatch of 10 generating units

The fuel cost of units 8 and 9 is very high, and the dispatch from these two units would significantly increase the total operation cost. The fuel costs of units 7 and 10 are the highest amongst all the generating units. Therefore, these two units are considered as reserve for unintentional outages or contingencies.

Table 3: Unit commitment schedule of 10 unit 24-h system

Unit (h)	1	2	3	4	5	6	7	8	9	10
1	1	1			1					
2	1	1			1					
3	1	1			1					
4	1	1			1					
5	1	1			1					
6	1	1		1	1					
7	1	1		1	1					
8	1	1		1	1					
9	1	1	1	1	1					
10	1	1	1	1	1	1				
11	1	1	1	1	1	1		1		
12	1	1	1	1	1	1		1	1	
13	1	1	1	1	1	1				
14	1	1	1	1	1					
15	1	1	1	1	1					
16	1	1	1	1	1					
17	1	1	1	1	1					
18	1	1	1	1	1					
19	1	1	1	1	1					
20	1	1	1	1	1	1				
21	1	1	1	1	1	1				
22	1	1		1		1				
23	1	1								
24	1	1								

The hourly operation cost of units is depicted in Table 4 and Fig. 5. Fig. 5 shows that the curve of the total cost follows the variation of the hourly demand power given in Fig. 3.

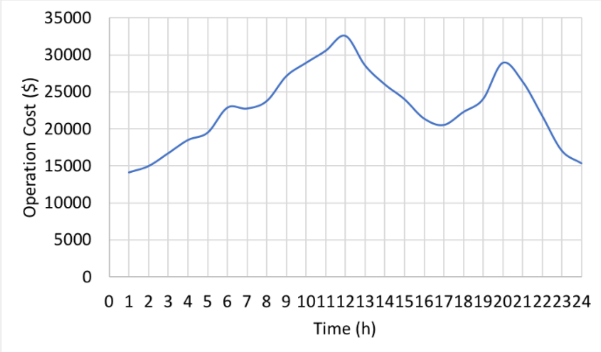

Fig. 5: Hourly operation cost of 10 generating units

4. Conclusion

Economic dispatch and unit commitment techniques are used in this paper to evaluate the minimum operation cost of a power network. MILP is used to express the cost function of the generating units while taking into consideration the technical constraints such as the min/max power, ramping up/down and minimum up/down time. The quadratic cost functions of the generating units are replaced with an equivalent piecewise function to satisfy programing purposes. A power network of 10 generating units is used to illustrate the effectiveness of the optimization method. The generating units were operated according to their operation characteristics with the minimum operation cost and without affecting the energy balance of the system. The optimal schedule of the unit commitment was presented to demonstrate the on/off status of the generating unit on an hourly basis.

Table 4: Fuel cost, start-up cost and total operation cost ($) of 10 unit 24 h system

Unit (h)	1	2	3	4	5	6	7	8	9	10	Fuel Cost	Start-up cost	Total
1	8426	4725			946						14096.87		14097
2	8426	5592			946						14964.1		14964
3	8426	7330			946						16701.47		16701
4	8426	8809			1244						18478.99		18479
5	8426	8809			2245						19480.35		19480
6	8426	8809		2846	1643						21724.48	1120	22844
7	8426	8809		2846	2649						22730.33		22730
8	8426	8809		2846	3664						23744.94		23745
9	8426	8809	2878	2846	3054						26013.12	1100	27113
10	8426	8809	2878	2846	3704	1904					28568.08	340	28908
11	8426	8809	2878	2846	3704	2178		1649			30490.9	60	30551
12	8426	8809	2878	2846	3704	2178		2093	1567		32501.66	60	32562
13	8426	8809	2878	2846	3704	1904					28568.08		28568
14	8426	8809	2878	2846	3054						26013.12		26013
15	8426	8809	2878	2846	1045						24004.37		24004
16	8426	6287	2878	2846	946						21383.6		21384
17	8426	5419	2878	2846	946						20515.33		20515
18	8426	7156	2878	2846	946						22252.24		22252
19	8426	8809	2878	2846	1045						24004.37		24004
20	8426	8809	2878	2846	3704	1904					28568.08	340	28908
21	8426	8809	2878	2846	2649	818					26426.54		26427
22	8426	8809		2846		1722					21803.27		21803
23	8426	8635									17060.86		17061
24	8426	6895									15321.29		15321
								Total			545416.5	3020	548436

List of symbols

C_T Total fuel cost
N Number of units
T Number of hours
P_i^t Output power in MW of unit *i* at time *t*
S_i^t Start-up cost of unit *i* at time *t*
D_i^t Shut-down cost of unit *i* at time *t*

u_i^t Status of unit i at time t (1 for ON and 0 for OFF)
S_{hi} Hot start-up cost of unit *i*
S_{ci} Cold start-up cost of unit *i*
$T_{i,ON}^t$ Duration in hour for which the unit i is continuously ON at time t
$T_{i,OFF}^t$ Duration in hour for which the unit i is continuously OFF at time t
T_i^D Minimum down time in hour of unit *i*
T_i^U Minimum up time in hour of unit *i*
T_i^C Cold start-up time in hour of unit *i*
P_D^t Total load in MW at time t
P_L^t Total system losses in MW at time t
$P_i^{\min}$ Minimum generation of unit *i*
$P_i^{\max}$ Maximum generation of unit *i*
SR^t System spinning reserve at time t

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